Indefinite Integration (IC-1)

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$$\int f(x)\,dx = g(x) + c \quad \text{if} \quad g'(x) = f(x)$$

Formulae

① $$\int (ax+b)^n\,dx = \frac{(ax+b)^{n+1}}{(n+1)a} + c, \quad n \neq -1$$

Sol. $$\int \sqrt{x}\,dx = \frac{2}{3}x^{3/2} + c$$

$$\int \frac{dx}{\sqrt{x}} = 2\sqrt{x} + c$$

② $$\int \frac{dx}{ax+b} = \frac{\ln|ax+b|}{a} + c$$

③ $$\int \sin(ax+b)\,dx = -\frac{\cos(ax+b)}{a} + c$$

④ $$\int \cos(ax+b)\,dx = \frac{\sin(ax+b)}{a} + c$$

⑤ $$\int \sec^2(ax+b)\,dx = \frac{\tan(ax+b)}{a} + c$$

⑥ $$\int \operatorname{cosec}^2(ax+b)\,dx = \frac{-\cot(ax+b)}{a} + c$$

⑦ $$\int \sec(ax+b)\tan(ax+b)\,dx = \frac{\sec(ax+b)}{a} + c$$

⑧ $$\int \operatorname{cosec}(ax+b)\cot(ax+b)\,dx = \frac{-\operatorname{cosec}(ax+b)}{a} + c$$

⑨ $$\int \tan(ax+b)\,dx = -\frac{\ln|\cos(ax+b)|}{a} + c \text{ or}$$

$$\frac{\ln|\sec(ax+b)|}{a} + c$$

⑩ $$\int \cot(ax+b)\,dx = \frac{\ln|\sin(ax+b)|}{a} + c \text{ or } -\frac{\ln|\operatorname{cosec}(ax+b)|}{a} + c$$

(11) $\int \sec(ax+b)\,dx = \dfrac{\ln|\sec(ax+b)+\tan(ax+b)|}{a} + c$

or $\dfrac{1}{a}\ln\left|\tan\left(\dfrac{\pi}{4}+\dfrac{ax+b}{2}\right)\right| + c$

Col.

$\int \sec x\,dx = \ln|\sec x+\tan x| + c$

or $\ln\left|\tan\left(\dfrac{\pi}{4}+\dfrac{x}{2}\right)\right| + c$

(12) $\int \operatorname{cosec}(ax+b)\,dx = -\dfrac{1}{a}\ln|\operatorname{cosec}(ax+b)+\cot(ax+b)| + c$

or

$= -\dfrac{1}{a}\ln\left|\cot\left(\dfrac{ax+b}{2}\right)\right| + c$

$\int \operatorname{cosec} x\,dx = \ln|\operatorname{cosec} x - \cot x| + c$

or $\ln\left|\tan\dfrac{x}{2}\right| + c$

(13) $\int e^{ax+b}\,dx = \dfrac{e^{ax+b}}{a} + c$

Col. $\int K^{ax+b}\,dx = \dfrac{K^{ax+b}}{a\ln(K)} + c$

(Desired)

(14) $\int \dfrac{dx}{(ax+b)(cx+d)} = \dfrac{1}{ad-bc}\ln\left|\dfrac{ax+b}{cx+d}\right|$

(15) $\int \dfrac{ax+b}{cx+d}\,dx = \dfrac{a}{c}x + \dfrac{bc-ad}{c^2}\ln|cx+d|$

$\left[\dfrac{dx}{\text{quadratic}}\right]$

(16) $\int \dfrac{dx}{x^2+a^2} = \dfrac{1}{a}\tan^{-1}\left(\dfrac{x}{a}\right) + c$

(17) $\int \dfrac{dx}{x^2-a^2} = \dfrac{1}{2a}\ln\left|\dfrac{x-a}{x+a}\right| + c$

(18) $\int \dfrac{dx}{a^2-x^2} = \dfrac{1}{2a}\ln\left|\dfrac{a+x}{a-x}\right| + c$

$\left[\dfrac{dx}{\sqrt{\text{quadratic}}}\right]$

(19) $\int \dfrac{dx}{\sqrt{x^2+a^2}} = \ln\left(x+\sqrt{x^2+a^2}\right) + c$

(20) $\int \dfrac{dx}{\sqrt{x^2-a^2}} = \ln\left(x+\sqrt{x^2-a^2}\right) + c$

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(21) $\int \frac{dx}{\sqrt{a^2-x^2}} = \sin^{-1}\left(\frac{x}{a}\right) + c$

(22) $\int \frac{dx}{x\sqrt{x^2-a^2}} = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) + c$ or

$-\frac{1}{a}\operatorname{cosec}^{-1}\left(\frac{x}{a}\right) + c$

[$\sqrt{\text{Quadratic}}\, dx$]

(23) $\int \sqrt{x^2+a^2}\, dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\ln\left(x+\sqrt{x^2+a^2}\right) + c$

(24) $\int \sqrt{x^2-a^2}\, dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left(x+\sqrt{x^2-a^2}\right) + c$

(25) $\int \sqrt{a^2-x^2}\, dx = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^{-1}\left(\frac{x}{a}\right) + c$

(26) $\int \sin^{-1}x\, dx = x\sin^{-1}x + \sqrt{1-x^2} + c$ [Derived]

(27) $\int \tan^{-1}x\, dx = x\tan^{-1}x - \frac{1}{2}\ln(1+x^2) + c$

(28) $\int \ln x\, dx = x\ln|x| - x + c$

(29) $\int x^x(1+\ln x)\, dx = x^x + c$

(30) $\int e^{ax}\sin(bx+c)\, dx = \frac{e^{ax}}{a^2+b^2}\left[a\sin(bx+c) - b\cos(bx+c)\right]$

$\int e^{ax}\cos(bx+c)\, dx = \frac{e^{ax}}{a^2+b^2}\left[a\cos(bx+c) + b\sin(bx+c)\right]$

Col. $\int e^x \sin x\,dx = \frac{e^x}{2}(\sin x - \cos x) + c$

$\int e^x \cos x\,dx = \frac{e^x}{2}(\cos x + \sin x) + c$

Methods of Integration

Type (0) Formulae Based

Type-1 Substitution $f(x) = t,\ f'(x)dx = dt$

(A) $\int (f(x))^n f'(x)\,dx = \frac{(f(x))^{n+1}}{n+1} + c \quad , n \neq -1$

(B) $\int \frac{f'(x)}{f(x)}\,dx = \ln|f(x)| + c$

(C) $\int \frac{f'(x)}{\sqrt{f(x)}}\,dx = 2\sqrt{f(x)} + c$

Type-2 $\int \frac{dx}{ax^2+bx+c}$ or $\int \frac{dx}{\sqrt{ax^2+bx+c}}$ or $\int \frac{lx+m}{ax^2+bx+c}\,dx$

or $\int \frac{lx+m}{\sqrt{ax^2+bx+c}}\,dx$ or $\int \frac{p(x)}{ax^2+bx+c}\,dx$

$= \int q(x)\,dx + \int \frac{lx+m}{ax^2+bx+c}\,dx$

$ax^2+bx+c \overline{)\ p(x)\ }\ q(x)$, $lx+m$ Rem.

In all these you need to apply

Completing the square technique in denominator &

use appropriate formula.

Also $lx+m = A(ax^2+bx+c)' + B$ i.e. $A(2ax+b) + B$

& write $I = I_1 + I_2$

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Type 3

A) $\int \frac{dx}{a\sin^2x + b\cos^2x + c\sin x\cos x + d}$

$\div$ by $\cos^2 x$ in Numerator & Denom.

then $\tan x = t$, $\sec^2 x\,dx = dt$

B) $\int \frac{dx}{a\sin x + b\cos x + c}$

$\sin x = \frac{2\tan\frac{x}{2}}{1+\tan^2\frac{x}{2}}$, $\cos x = \frac{1-\tan^2\frac{x}{2}}{1+\tan^2\frac{x}{2}}$

$\tan\frac{x}{2} = t$, $\sec^2\frac{x}{2}\,dx = 2dt$

C) $\int \frac{l\sin x + m\cos x + n}{a\sin x + b\cos x + c}\,dx$

$l\sin x + m\cos x + n = A(a\cos x - b\sin x) + B(a\sin x + b\cos x + c) + C$

Type 4 Standard Integrals

1) $\int \frac{x^2+1}{x^4+1}\,dx$

$\div\, x^2$

$x - \frac{1}{x} = t$

2) $\int \frac{x^2-1}{x^4+1}\,dx$

$\div\, x^2$

$x + \frac{1}{x} = t$

3) $\int \frac{1}{x^4+1}\,dx$ $\quad \frac{① - ②}{2}$

4) $\int \frac{x^2}{x^4+1}\,dx$ $\quad \frac{① + ②}{2}$

5) $\int \sqrt{\tan x}\,dx$

$\tan x = t^2$

or $\int \sqrt{\cot x}\,dx$

$\cot x = t^2$

(6) $$\int \frac{dx}{\underset{L_1}{(ax+b)}\underset{\sqrt{L_2}}{\sqrt{cx+d}}}$$ $cx+d=t^2$ L_2

(7) $$\int \frac{dx}{\underset{Q}{(ax^2+bx+c)}\underset{\sqrt{L}}{\sqrt{lx+m}}}$$ $lx+m=t^2$

(8) $$\int \frac{dx}{\underset{L}{(lx+m)}\underset{\sqrt{Q}}{\sqrt{ax^2+bx+c}}}$$ $\overset{L}{lx+m}=\frac{1}{t}$

(9) $$\int \frac{dx}{\underset{Q_1}{(ax^2+b)}\underset{\sqrt{Q_2}}{\sqrt{cx^2+d}}}$$ $x=\frac{1}{t}$

(10) $$\int f\left(x+\sqrt{x^2+a^2}\right)dx$$ $x+\sqrt{x^2+a^2}=t$

Type.5 Integration By Parts (Product Rule)

$$\int \underset{\text{I}}{u}\,\underset{\text{II}}{v}\,dx = u\int v\,dx - \int\left(u'\int v\right)dx$$

$$\text{or}\quad \text{I}\int \text{II} - \int\left(\text{I}'\int \text{II}\right)$$

where I & II decided by ILATE Rule

- I → Inverse Trigo.
- L → logarithmic
- A → Algebraic
- T → Trigo
- E → Exponential

function which came first in ILATE Rule is taken as 'I'

Type 6 A) $\int e^x (f(x) + f'(x)) dx$

$$= e^x f(x) + C$$

B) $\int e^x (f(x) - f''(x)) dx$

$$= \frac{e^x}{2} (f(x) - f'(x)) + C$$

Type-7 $\int \sqrt{ax^2+bx+c}\, dx$ or $\int (lx+m)\sqrt{ax^2+bx+c}\, dx$

$ax^2+bx+c \rightarrow$ completing the square technique & Apply formula

If $lx+m = A(2ax+b) + B$

$$I = I_1 + I_2$$

Type. 8 Irrational functions

A) $\int f((ax+b)^{1/l}, (ax+b)^{1/m}, (ax+b)^{1/n}) dx$

$l, m, n \in N$

or

$\int f(\sqrt[l]{ax+b}, \sqrt[m]{ax+b}, \sqrt[n]{ax+b}, ..) dx$

LCM(l, m, n)

let $ax+b = t$

B) $\int x^{\frac{1}{m}} (a + bx^{\frac{1}{n}})^{\frac{1}{p}} dx$, Try one of the following substitutions

$m, n, p \in N$

(i) $x = t^{LCM(m,n)}$

(ii) $a + bx^{1/n} = t^p$ (iii) $ax^{-1/n} + b = t^p$

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Type 9 Partial fraction Technique

$$\int \frac{p(x)}{q(x)}\,dx$$, $p(x)$ & $q(x)$ are polynomials

If degree of $p(x) \geq$ degree of $q(x)$

$$q(x)\overline{)\,p(x)\,}\,(\,l(x)$$
$$\frac{=}{R(x)}$$

$$\int \frac{p(x)}{q(x)} = \int l(x) + \int \frac{R(x)}{q(x)}$$

factorize $q(x)$

For each $(x-\alpha)^n$ in $q(x)$ write n partial terms

$$\frac{A_1}{x-\alpha} + \frac{A_2}{(x-\alpha)^2} + \cdots + \frac{A_n}{(x-\alpha)^n}$$

For each (quadratic) $(ax^2+bx+c)^m$

$$\frac{B_1x+C_1}{ax^2+bx+c} + \frac{B_2x+C_2}{(ax^2+bx+c)^2} + \cdots + \frac{B_mx+C_m}{(ax^2+bx+c)^m}$$

$$\frac{R(x)}{q(x)} = \frac{A_1}{x-\alpha} + \frac{A_2}{(x-\alpha)^2} + \cdots + \frac{B_1x+C_1}{(ax^2+bx+c)} + \cdots$$

find A_i, B_i, C_i by comparing like powers of x^i

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Type 10: Reduction Formulae

(A) $I_n = \int \sin^n x\, dx = -\frac{\sin^{n-1} x \cos x}{n} + \frac{n-1}{n} I_{n-2}$

(B) $I_n = \int \cos^n x\, dx = \frac{\cos^{n-1} x \sin x}{n} + \frac{n-1}{n} I_{n-2}$

(C) $I_n = \int \tan^n x\, dx = \frac{\tan^{n-1} x}{n-1} - I_{n-2}$

(D) $I_n = \int \cot^n x\, dx = -\frac{\cot^{n-1} x}{n-1} - I_{n-2}$

(E) $I_n = \int \sec^n x\, dx = \frac{\sec^{n-2} x \tan x}{n-1} + \frac{n-2}{n-1} I_{n-2}$

(F) $I_n = \int \operatorname{cosec}^n x\, dx = -\frac{\operatorname{cosec}^{n-2} x \cot x}{n-1} + \frac{n-2}{n-1} I_{n-2}$

(G) $I_n = \int_0^{\pi/2} \sin^n x\, dx = \int_0^{\pi/2} \cos^n x\, dx = \frac{n-1}{n} I_{n-2}$

(H) $I_n = \int_0^{\pi/4} \tan^n x\, dx = \frac{1}{n-1} - I_{n-2}$